In conclusion, I would express my obligations to Mr. W. Esson for assistance in the mathematical portion of the paper, and to my colleagues for suggestions made in the course of the investigations.

VII. "On the Theory of Electrodynamics, as affected by the Nature of the Mechanical Stresses in Excited Dielectrics." By J. Larmor, M.A., Fellow of St. John's College, Cambridge. Communicated by Professor J. J. Thomson, F.R.S. Received April 25, 1892.

1. A theory of electrodynamics was first precisely developed by Maxwell, which based the phenomena on Faraday's view of the play of elasticity in a medium, instead of the conception of action at a distance, by means of which the mathematical laws had been primarily evolved. The electromotive equations of Maxwell however involve nothing directly of the elastic structure of this medium, which remains wholly in the background. They involve simply the assumption of a displacement across dielectrics with such properties as to make all electric currents circuital; all the equations of Ampère and Neumann for closed or circuital currents have then a universal validity, and no further hypothesis is required for the full development of the subject.

The theory was next discussed by Helmholtz in his memoirs on electrodynamics, in a way which took direct advantage of the picture of a polarised dielectric supplied by Mossotti's adaptation of the Poisson theory of induced magnetisation. Stated absolutely, this simply builds upon the assumption that at ecah point in the excited dielectric there is something which has the properties of a current element (electric transfer or displacement), which is represented both in direction and magnitude by the electric force at the point multiplied by a constant factor; no more general starting point seems possible for an isotropic dielectric. The development of this hypothesis, exactly on the analogy of a similar discussion with the Poisson-Mossotti phraseology in a previous paper,* leads to the necessity of recognising the existence of absolute electric charges on the faces of an excited condenser; so that the exciting current causes the accumulation of these charges, and therefore is not circuital or solenoidal. This defect of circuital character however practically disappears in the limiting case when the constant ratio of the polarisation to the electric force is extremely great; and then the theory becomes a concrete illustration of the general statements of Maxwell with respect to electric displacement.

* "The Theory of Electrodynamics," 'Roy. Soc. Proc.,' 1891.

It was shown in the paper above referred to that this hypothesis, adopted by Helmholtz, led by itself—without any necessity for further assumptions which its author introduced on various grounds—to the undulatory propagation of electromotive disturbances across dielectric media, with the same transverse type of waves as constitute light. There will usually be, in addition, a disturbance of a *quasi*-compressional character, which, on the more special hypothesis of Helmholtz, is also propagated as a wave of permanent type, but with different velocity. The electric undulations of transverse type have been detected by Hertz; and the balance of evidence, from the experiments of different authors, seems to point to the conclusion that their velocities in different media are inversely as the square roots of the specific inductive capacities. Should this be fully verified, it would follow demonstratively that the Helmholtz hypothesis must be restricted to the special form which represents the Maxwell displacement theory; and the general equations of electrodynamics, or rather the electromotive part of them, will be definitely established.

2. The object here proposed is to pass on from the electromotive to the ponderomotive properties of the electric field, and examine whether the latter lend any strength to the conclusions derived from the former. Instead of a kinetic phenomenon like undulatory propagation, we shall now consider the static phenomena of the stress produced in the material of a dielectric by its excitation; and, to avoid the complexity, both optical and mechanical, introduced by the elasticity of solids, we shall consider solely liquid dielectrics, on which a very valuable series of experiments has been made by Quincke.* The mechanical stress in a fluid depends on one variable, the intensity of the hydrostatic pressure, and therefore may be connected immediately with the distribution of the energy in the medium, by means of the principle of work.

The arguments for the actual existence of a stress of the Maxwell type may be exhibited in a synthetical manner as follows:—Consider a condenser formed by two closed conducting sheets, one inside the other; and imagine the equipotential surfaces to be traced in the excited fluid dielectric between them. It is a matter of experimental knowledge that there is a traction on each face, acting inwards, and equal, at any rate approximately, to $KF^2/8\pi$ per unit surface, where F is the electric force. Now the electric potential and therefore the state of the dielectric fluid, will be in no wise altered if we imagine a very thin stratum along one of the equipotential closed surfaces to become conducting. There will therefore be a normal traction given by the same formula, on each element of area of this surface. If this traction is an affair transmitted across the medium, the transmitting stress must be a tension $KF^2/8\pi$ along the lines of force. To form an

* 'Wiedemann's Annalen,' vol. 19, 1883.

opinion as to whether a medium transmitting stress in this way could be imagined, let us suppose the dielectric divided into thin layers, like those of an onion, by much thinner conducting sheets, which coincide with the equipotential surfaces. The potential will not thereby be altered; if we run a tube of force across the dielectric, equal and opposite charges will reside on the portions of the two faces of each sheet intercepted by it. The layers of dielectric will be electrically independent of each other, being separated by conducting layers. Each dielectric layer will, therefore, form a condenser, and the energy of its electrification per unit surface will be $K(\delta V)^2/8\pi t$, or $KF^2 t/8\pi$, where t is the thickness at the point, and δV the difference of potential between the faces; that is, there will be a distribution of energy $KF^2/8\pi$ per unit volume. The resultant traction on both the equal and opposite charges, each σ per unit area, on the two faces of a layer of dielectric, will be normal to the layer, and equal to $\frac{1}{2}\sigma(dF/dn)\delta n$ per unit surface; now, by Green's form of Laplace's equation, $\frac{dF}{dn} = F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$, where R_1, R_2 are the radii of principal curvature of the sheet; thus the traction is $\frac{F^2\delta n}{8\pi}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$. By the theorem of surface tension, this normal traction will produce and be balanced by a uniform surface tension along the sheet, of intensity $F^2\delta n/8\pi$, or $F^2/8\pi$ per unit thickness. In this laminated medium, owing to the attraction across the layers of very small thickness, we have thus set up a tension $KF^2/8\pi$ along the lines of force, which by reaction on the medium produces a pressure uniform in all directions round the lines of force, of the same numerical value. Or, again, we might, following Maxwell, postulate that the stress system in the medium must be symmetrical round the lines of force, and deduce, by the condition of internal equilibrium, that the tension and pressure of which it must thus consist are equal. A spherical system will form a simple illustration, capable of elementary treatment.

The fact that the surface of a dielectric liquid like petroleum is raised up by attraction, towards an electrified body brought near it, also affords evidence that this tension must exist. Consider two horizontal condenser plates, one inside the petroleum and the other over its surface in air. When the condenser is charged, the surface of the fluid rises between the two plates. There must, therefore, be some traction acting on it upwards to sustain it against gravity. The intensity of this traction is, in fact, according to Maxwell's law, $\frac{F^2}{8\pi}-\frac{K}{8\pi}\left(\frac{F^2}{K}\right)$, that is, $\frac{F^2}{8\pi}\left(1-\frac{1}{K}\right)$, where F is the electric force in the air; being positive, it acts upwards, in accordance with the actual phenomenon. Without the assistance of a traction of this kind, the fact would be unexplained, unless by assuming, with Helmholtz,

the existence of a *quasi*-magnetic polarisation of the elements of the medium; that would lead, on the interface between two media, to an uncompensated sheet of poles of density $\frac{K_1-1}{4\pi}F_1-\frac{K_2-1}{4\pi}F_2$, subject to a mean force $\frac{1}{2}(F_1+F_2)$; so that, as $K_1F_1 = K_2F_2$, the traction would be $(F_1^2-F_2^2)/8\pi$, or, in the example chosen above, $\frac{F^2}{8\pi}\left(1-\frac{1}{K^2}\right)$. The discrepancy between these values might, perhaps, be amenable to experiment; but I find, on trial, that the difficulty of obtaining a clean unelectrified surface is not easily overcome.

The observation of Faraday, that short filaments of silk or other dielectric material suspended in a fluid dielectric set themselves along the lines of force when it is excited, is also evidence of actual internal polarisation related to the lines of force.

For the case of a fluid, the Faraday-Maxwell stress is made up of a hydrostatic pressure, $KF^2/8\pi$, which is consistent with simple fluidity, together with a tension $KF^2/4\pi$ along the lines of force, which requires for its maintenance qualities other than those of isotropic mechanical fluidity.

3. The polarisation theory, in the form of Mossotti and Helmholtz, which locates part of the electrification in a displacement existing in the elements of the dielectric, and part of it in an absolute electric charge situated on the plates of the condenser the cause of that displacement, is the representation of a wider theory which supposes the electrostatic energy to be in part distributed through the dielectric as a volume density of energy, and in part over the plates as a surface density. If experiment show that the latter part is null, we are precluded from imagining any superficial change on the plates which has a separate existence, and is not merely the aspect at one end of the displacement across the volume of the dielectric. We shall find reason to conclude that there is no superficial part in the distribution of energy; this would carry the result that the excitation of a condenser consists in producing a displacement across the dielectric which just neutralises the charge conducted to the plates; it would also carry the result that all currents, whether in conductors or in dielectrics, must flow in complete circuits, and would therefore confirm the Maxwell theory of electrodynamics.

The conclusion that the location of all the electrostatic energy in the dielectrics involves that all currents flow in complete circuits seems of importance sufficient to justify a few remarks on the nature of the evidence on which it is based. The only precise notion or illustration of the nature of the dielectric polarisation which has yet been advanced is that of Poisson, which has been at various times used and developed by Mossotti, Faraday, Thomson, and Helmholtz. It might be held merely on general grounds that it gives a correct

formal view of the phenomenon, though the dynamical machinery of which it represents the action is quite unknown. But this presumption is very much strengthened by the fact that the displacement or polarisation is known to present qualitatively the properties of a true current, and also that the theory of dielectric propagation developed from this basis presents all the general analogies to light propagation that have been experimentally confirmed by Hertz and others. Taking it then that dielectric polarisation is formally of this type, the absence of a sensible absolute exciting charge on the bounding plates will show that it must be, so to speak, self-excited, that it is of the formal character of a displacement, of something pushed across from one plate towards the other like an incompressible substance.

4. Let us confine our attention for definiteness to the case of two metallic plates immersed horizontally near together in an extended mass of a fluid dielectric, so as to form a condenser. The traction T per unit area on the upper plate may by this arrangement be directly weighed. Suppose that there is a small aperture in the centre of the upper plate through which a volume of a different dielectric, say, a bubble of air, may be introduced between the plates, so as to form a flat cylinder co-axial with the plates and bounded above and below by them. The extra pressure P in this air bubble, when the condenser is excited, may be measured by a manometer in connexion with it, and it will give the means of determining the pressure in the surrounding liquid dielectric. This arrangement describes in fact the plan of Quincke's experiments.

At a point in the dielectric where the electric force is F, the electric pressure will be proportional to F^2, say, A_2F^2 for the liquid, and A_1F^2 for the air. The air column in the manometer tube would thus be in internal equilibrium with an electric pressure A_1F^2 next the liquid and null at the manometer end. We might at first glance infer that under these circumstances the pressure A_1F^2 is not indicated by the manometer, which would thus record simply the electric pressure in the liquid. But this air pressure is an internal stress; the equilibrium of any section of the air column requires, in order to maintain it, the electric pressure against it of the air on the other side of that section; therefore, the indication of the manometer really gives the differential effect $A_2F_2^2-A_1F_1^2$.

Let now the volume energy of the electrification be C_2F^2 in the liquid, and C_1F^2 in the air, each per unit volume; and let the energies of such real surface electric distributions as might exist on the plates in contact with these dielectrics be Σ_2 and Σ_1 respectively, each per unit surface of both plates. These surface energies would involve in their expression the electric potential as well as the force. We may apply the principle of virtual work to determine the relations between the quantities thus defined.

Suppose that the distance between the plates, with no air bubble introduced, is slowly increased from c to $c+\delta c$ against the tension T_2 which tends to draw them together. The work done against T_2 in raising the upper plate is the only source of the additional energy of the system which appears when that separation is effected. Now the value of F is not thereby altered, as the electrification remains constant. The volume energy contained in a cylinder of the liquid of unit sectional area is therefore changed from C_2F^2c to $C_2F^2c\,(1+\delta c/c)$. The corresponding surface energy is changed from Σ_2 to $\Sigma_2+\frac{d\Sigma_2}{dc}\delta c$. Therefore, by the principle of work, $T_2 = C_2F^2+\frac{d\Sigma_2}{dc}$.

Now suppose a large air bubble introduced between the plates; and suppose its volume to be increased by δv owing to a virtual displacement produced by pressing in more air. The virtual work $P\delta v$ so done must be equal to the increase of the internal energy of the system due to the displacement. This increase may be calculated from the energy function of the actual conformation, not of the displaced position, as internal equilibrium subsists; and all considerations as to change of intrinsic energy may thus be evaded.* For a bubble of small dimensions its surface would, in fact, be increased by the supposed displacement, and so there would be an increase of intrinsic capillary energy; with surface tension τ, radius a, and a semicircular meniscus πc, there would be an increase of amount $2\pi^2 c\tau\delta a$, that is $\pi\tau c\delta v/a$. Again, the electric forces F_1 and F_2 in the two media, measured not close up to the meniscus, are equal, because the plates are each at uniform potential. Close to the centre of the meniscus they are both tangential to it, and must also be equal on the two sides. But along the slope of the meniscus they are oblique to it and are unequal, their relation being determined on the ordinary theory by the continuity of the normal component induction and of the tangential component force. Thus the difference of electric pressure across the meniscus will vary from point to point of it, and its form will, therefore, be slightly altered by the electrification. It follows from the observations of Quincke and others that its capillary constant will not thereby be altered, as was to be expected because there is no extra supply of molecular surface energy. There is also the alteration of intrinsic energy due to the fact that the expansion of the air bubble alters the electric force. But, according to the principles just stated, these changes of intrinsic energy balance each other, because all the parts of the medium are in internal equilibrium. We may therefore consider the annular mass between two ideal coaxial cylindric surfaces at a distance from the meniscus, one in the

* For applications of this principle, *cf.* Helmholtz, 'Wied. Ann.,' vol. 13, p. 388; and Kirchhoff, 'Wied. Ann.,' vol. 24, p. 57.

bubble and one in the liquid, and reckon the change in the energy contained in the space originally occupied by this annulus, when it receives a small displacement outwards from the axis under the action of the manometer pressure P. That change is $-(C_2-C_1)F^2\delta v$, where F is the electric force at a distance from the meniscus. Therefore, by the principle of virtual work, $P\delta v-(C_2-C_1)F^2\delta v = 0$, so that $P = (C_2-C_1)F^2$.

The value of the traction between the two plates in air is given by this formula as $T_1 = C_1F^2+\dfrac{d\Sigma_1}{dc}$; this must, therefore, be the same as the well-ascertained experimental value $F^2/8\pi$. Now the experiments of Quincke and others on liquid dielectrics have given reason to believe that, within the limits of experimental uncertainty due to want of purity of the materials and other causes, $T_2 = P+F^2/8\pi$. It follows that we must have $d\Sigma_2/dc = d\Sigma_1/dc$; that is, $d\Sigma/dc$ must be the same for all media, which is physically consistent only with the non-existence of this surface energy, unless we can suppose it to be the energy of an action at a distance which is independent of the intervening medium altogether.

The argument may also be expressed somewhat differently as follows:—The plates of the condenser being supported independently, the existence of an extra pressure on the dielectric when the condenser is excited shows that part, at any rate, of the electric energy resides in the dielectric. That part must, on any view, either of action by contact or of *quasi*-magnetic polarisation, be proportional to the square of the electric force at the point, which is in fact confirmed by the experiments of Silow on a quadrant electrometer with its needle immersed in a dielectric liquid filling the quadrants. If this were the whole of the electric energy, the traction between the plates would be equal to the hydrostatic pressure in the dielectric, or at most differ from it by an amount which would be the same for all media.* If this were only part of the electric energy, the difference would depend on the other superficial part. The experiments show that for a large number of liquids the difference is very nearly the same, so that if, after Quincke, we suppose it to be null for air or vacuum, it is null for all the others.† Hence either the superficial energy must be absolutely independent of the nature of the dielectric

* *Cf.* J. Hopkinson, 'Roy. Soc. Proc.,' 1886, p. 453, for the use of a similar argument in the converse manner, to show that the tension and pressure must be equal; but in it the energy of the polarisation of the medium is apparently not sufficiently traced.

† The results of Quincke are calculated so as to give values for K_p, the inductive capacity deduced from experiments on fluid pressure, and K_s, the inductive capacity deduced from experiments on the traction between the plates, on the assumption that the stress is of the Faraday-Maxwell type. The following examples show the order of magnitude of the discrepancies—

or else it must be non-existent. The precise logical statement of Quincke's results is, in fact, that the difference between the electric stress in a ponderable fluid dielectric K and the electric stress in a vacuum, in a field of force F, consists of a tension $\frac{K-1}{8\pi}F^2$ along the lines of force, combined with a pressure of equal amount in all directions at right angles to them; and this is consistent with a distribution of polarisation energy in the fluid, added to the electric energy for a vacuum with the same intensity of force, but not entering into combination with it.

Now the propagation of electrical waves across air or vacuum shows that even then, when there is no ponderable dielectric present, there must be a store of statical energy in the dielectric; and this fact appears to remove the only explanation which seems assignable for the division of the energy into two parts, one located in the dielectric, and the other located on the plates and absolutely independent of the dielectric, viz., that the latter might be the energy of a direct action across space which is not affected by the dielectric. The experimental facts, therefore, so far tend to the conclusion that at any rate the basis of electrical theory is to be laid on Maxwell's lines, with a reservation for possible modification in the form of residual corrections, but not for change of principle.

A theory has been developed by Helmholtz for fluids, and by Kirchhoff, following him, for solid dielectrics, in which slight residual differences between the intensities of the tension and pressure may be accounted for on the supposition that the inductive capacity, instead of being constant, is a function of the electric force. This theory is primarily expounded in terms of a polarisation scheme, and in so far is subject to the remarks of the next section; but it may in the end be based, as Helmholtz suggested, on the principle of energy applied with the aid of the ascertained form of the characteristic equation of the potential, treated as a condition of internal equilibrium. If we adopt the view that the difference to be explained has not certainly been detected,* this theory need not here be considered.

Some of the points in the general treatment given above will

	K_p.	K_s.
Ether	4·62	4·66
Carbon disulphide	2·69	2·75
Benzol	2·32	2·37
Turpentine	2·26	2·35
Petroleum	2·14	2·15

The chief difficulty seemed to be to avoid conduction, owing to want of purity of the dielectric fluid.

* *Cf.* Bos, "Inaugural Dissertation," abstracted in 'Philosophical Magazine,' February, 1891.

also be illustrated by the following brief discusion, which has special reference to the Mossotti-Helmholtz polarisation theory. In the course of it reasons will appear that even the special limit of that theory which coincides with Maxwell's as to form must be abandoned as inconsistent with the dynamical phenomena, in favour of a theory of pure contiguous action or strain of an incompressible æther.

Without entering here into detail as to the general characteristics of this kind of polarisation, it will suffice to point out some of its principal relations with regard to which misconception is easy, and also to point out the modifications which are necessitated in its usual form by the recognition of the discrete or molecular character of the polarised elements. In the Poisson theory of induced magnetism the magnetic potential is the potential not of the actual magnetism, but of the continuous volume and surface distributions of ideal magnetic matter which Poisson substitutes for it. The forces on a magnetic molecule are therefore not to be derived from it.* But if we imagine a very elongated cavity to be scooped out in the medium along the direction of magnetisation, and the molecule to be placed in the middle of the cavity, the forces of the remaining magnetised matter will be correctly derived from this potential. This part of the forcive will thus be derivable from a potential energy MF cos ϵ, where M is the moment of the molecule, F the resultant force derived from the magnetic potential, and ϵ the angle between their directions; we may thus consider a potential energy function IF cos ϵ per unit volume. We have to add to these forces the ones due to the rejected magnetic molecules which lay in the elongated cavity. Now the mutual action of contiguous magnetic molecules will be of the nature of a tension along the lines of magnetisation and a pressure at right angles to them, as Helmholtz remarked;† but these stresses will not necessarily be equal in intensity; nor will they represent the Faraday-Maxwell stress, since each component is proportional to the square of the coefficient of magnetisation, not to its first power. In a fluid medium these forces also must be derivable from an energy function, for otherwise the medium could not be in equilibrium; and the total potential energy per unit volume with its sign changed is equal to the fluid pressure. Thus in the polarised fluid the pressure is

$$\tfrac{1}{2}\mathrm{FI} + \tfrac{1}{2}\lambda\mathrm{I}^2,$$

that is,

$$\tfrac{1}{2}(\kappa + \lambda\kappa^2)\,\mathrm{F}^2.$$

* In estimating these forces it is not allowable to replace the molecule by its three components parallel to the axes in the usual manner. This procedure would lead to error if there are electric currents in the field. *Cf.* Maxwell, 'Electricity,' ed. 2, vol. 2, ch. xi, appendix 2, p. 262.

† 'Wiedemann's Annalen,' vol. 13, 1881, p. 388.

An actual illustration in which the term involving λ is of predominant importance is afforded by a bunch of iron nails hanging end to end from a pole of a magnet; the adjacent nails hang on to each other lengthways and repel each other sideways, while the action of non-adjacent ones is but slight.

In the electric polarisation theory the specific inductive capacity is $K = 1 + 4\pi\kappa$. The results of Quincke, above mentioned, after they had been corrected for an experimental oversight in the direct determinations of the values of K by experiments on capacity, in accordance with a suggestion made by Hopkinson,* made the electric pressure to be $KF^2/8\pi$, consistently within the limits of experimental error for fifteen different substances. Thus, even in the limiting Maxwell form of the theory, which takes the absolute numerical value of K to be very great, this theory would not fit with the experiments unless λ is zero. Even by the purely mathematical device of taking the polarised elements to be right solids closely packed together, it does not seen possible to evade this argument.

In an actual fluid polarised in the above manner each element might on the average be considered as lying at the centre of a cavity, a sort of sphere of action within which the other molecules in their motions do not approach it further. On averaging the positions of these surrounding molecules during their motions with respect to the one under consideration, we arrive at the conception of a continuous polarised medium with a cavity in it of the form of this sphere of action. If this cavity were an actual sphere, the value of λ would be $\frac{4}{3}\pi$; and for cavities not very greatly different from the spherical form, the alteration in this value would be insensible. Under no likely circumstances could the value of λ come to be zero.

Thus the limiting Helmholtz polarisation representation of an excited dielectric, though complete as regards electromotive properties, would appear to fail to include the static ponderomotive phenomena of electrification, and requires to be modified into some more continuous mechanism, such as an elastic displacement in an æther loaded with the molecules of the dielectric.

It may be well to remark that, on account of the extreme smallness of the magnetic coefficient κ for all fluids, its square is of no account in comparison, and therefore magnetic pressures are sufficiently represented by the simpler formula $\frac{1}{2}\kappa F^2$, by means of which Quincke has measured the magnetic constants of various fluid media.

5. The principal conclusions which have been arrived at are here enumerated.

(i.) It is shown from experimental results that the stress in an excited fluid dielectric between two condenser plates consists, at any rate to a first approximation, of a tension along the lines of force and

* 'Roy. Soc. Proc.,' 1886.

an equal pressure in all directions at right angles to them, superposed upon such stress as would exist in a vacuum with the same value of the electric force.

(ii.) It is shown from experiments that the numerical value of these additional equal tensions and pressures is, at any rate to a first approximation, $(K-1)F^2/8\pi$, where F is the electric force, and K the inductive capacity.

(iii.) Such a distribution of equal tension and pressure is the result of a uniform volume distribution of energy in the dielectric, irrespective of what theory is adopted as to its mode of excitation.

(iv.) If we consider the mode of excitation to be a *quasi*-magnetic polarisation of its molecules, the numerical magnitude of these stresses should be

$$\frac{K-1}{8\pi} F^2 \left(1+\lambda \frac{K-1}{4\pi}\right),$$

where λ is a coefficient which depends on the molecular discreteness of the medium, and is probably not very different from the value $\frac{4}{3}\pi$. A discrete polarisation of the molecules does not account for the stress, so far as this coefficient is concerned.

(v.) The stress which would exist in a vacuum dielectric is certainly due in part to a volume distribution of energy, as is shown by the propagation of electric waves across a vacuum. There is thus no reason left for assuming any part of it to be due to a distribution of energy on its two surfaces, acting directly at a distance on each other. There is therefore ground for assuming a purely volume distribution of energy in the vacuous space, leading to a tension $F^2/8\pi$ along the lines of force, and a pressure $F^2/8\pi$ at right angles to them.

(vi.) The *quasi*-magnetic polarisation theory rests on the notion of a dielectric excited by a surface charge on the plates, and therefore involves a surface distribution of energy, except in the extreme case when the absolute value of K is very great; in that case a slight surface charge produces a great polarisation effect, and in the limit the polarisation may be taken as self-excited. Thus the absence of a surface distribution of energy leads to Maxwell's displacement theory, in which all electric currents are circuital, and the equations of electrodynamics are therefore ascertained.

(vii.) It appears that even this limiting polarisation theory must be replaced, on account of the stress-formula in (iv), by some dynamical theory of displacement of a more continuous character.

6. We may perhaps attempt to form a more vivid picture of the interaction between æther and matter by following out the ideas of Lord Rayleigh's version of Young's theory of capillarity. We may conceive the compound medium, æther and matter, to consist of a very refined æthereal substratum, in which the molecular web of

matter is imbedded. The range of direct action between contiguous parts of the æther would be very small, and that between contiguous elements of matter large in comparison. There exist disturbances in which the matter-web is unaffected, its free periods being too slow to follow them: these are propagated with great velocity as light, or electrical radiations. There are other disturbances in which the matter-web is alone active; these are so slow that the æther can adjust itself to an equilibrium condition at each instant; they are propagated as waves of material vibration or sound waves.

When a dielectric is excited, we find ourselves in the presence of a strain of an æthereal origin somehow produced; it would relax on discharge of the system with the velocity of light. At an interface where one dielectric joins another, the æthereal conditions will somehow, owing to the nature of the connexion with the matter, only admit of a portion of the stress being transmitted across the interface; and there will thus be a residual traction on the interface which must, if equilibrium subsist, be supported by the matter-web, and be the origin of the stress which has been verified experimentally. Inside a conductor, the æther cannot sustain stress at all, so that the whole æthereal stress in the dielectric is supported by the surface of the matter-web of the conductor. At such interfaces the æthereal part of the distribution of energy in the medium will be discontinuous.

A formula has been given by Maxwell* for the intensity of the pressural force produced by electric undulations in the æther striking against a plate of conducting matter, a force which has apparently not been detected for the case of light-waves. If the notions here suggested have any basis, this force may likely be non-existent. For the pulsations of the æther at this surface may be so rapid as to prevent their energy being communicated to the matter-web of the conductor; and the energy will then be scattered and lost instead of appearing as energy of material stress. We may take as an illustration a stretched cord with equidistant equal masses strung on it, for which Lagrange showed that if the period of a disturbance imparted at one end exceeds a certain limit, the disturbance will not be transmitted into the cord, but will be eased off within a short distance of the point of application. And also in a manner which forms a more exact analogy, Sir G. Stokes has shown that the higher harmonics of a telegraph wire vibrating in the wind have their pulsations too rapid to get a grip on the air around them, and their note is therefore not transmitted.

This view would place the electrostatic and electrodynamic forces on matter on a lower plane, and in the case of rapid or sudden disturbance a more uncertain one, than the electromotive phenomena.

* 'Electricity,' § 793.

VIII. "On Current Curves." By Major R. L. HIPPISLEY, R.E. Communicated by Major MACMAHON, F.R.S. Received May 12, 1892.

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